Date 12/10/2023

Saathi

Chapter-7

Exercise-7.1

1. Total digits = 5

3 digit Numbers when rep. of digit is allowed

$= 5 \times 5 \times 5 = 125$ (Ans)

(ii) 3 digit numbers when rep. of digits is not allowed

$= 3 \times 4 \times 5 = 60$ (Ans)

2.

digits - 1, 2, 3, 4, 5, 6

Total digits = 6

Four even number 2, 4, 6 must be at unit place.

No. of three digit even numbers

$= 6 \times 6 \times 3$

$= 108$ (Ans)

3.

Total digits = 10

No letter can be repeated

$7 \times 8 \times 9 \times 10$

5040

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4. Total digits = 10

No. of possible 5 digit Telephone numbers starting with 67

$= 6 \times 7 \times 8$

$= 336$ (Ans)

5. Total no. of possibilities = 2

$2 \times 2 \times 2 = 8$

6. Total flags = 5

$5 \times 4 = 20$ (Ans)

Exercise - 7.2

1. Evaluate

(i) $8!$

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320$ (Ans)

(ii) $4! - 3!$

$4 \times 3 \times 2 \times 1 - 3 \times 2 \times 1$

$24 - 6 = 18$ (Ans)

2. Is $3! + 4! = 7!$?

L.H.S. :- $3 \times 2 \times 1 + 4 \times 3 \times 2 \times 1$

$6 + 24$

$= 30$ (Ans)

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R.H.S. $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

No $3! + 4! \neq 7!$

Q3- Compute $\frac{8!}{6! \times 2!}$

$\frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = 28$ (Ans)

Q4- If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x

$\frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8 \times 7 \times 6!}$

$\frac{1}{1} + \frac{1}{7} = \frac{x}{56}$

$\frac{7+1}{7} = \frac{x}{56}$

$8 \times 56 = 7x$

$\frac{8 \times 56^{8}}{7} = 64 = x$ (Ans)

Q5- Evaluate $\frac{n!}{(n-r)!}$ when

(i) $n=6$, $r=2$

$\Rightarrow \frac{6!}{(6-2)!}$

$\Rightarrow \frac{6 \times 5 \times 4!}{4!} \Rightarrow 30$ (Ans)

(ii) $n=9$, $r=5$

$\Rightarrow \frac{9!}{(9-5)!}$

$\Rightarrow \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} \Rightarrow 15120$ (Ans)

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Exercise - 7.3

1. How - - - - - - - - - - - - repeated?

Total digits = 9

3 digit no. when no repitition is allowed

$= 9 \times 8 \times 7$

$= 504$ (Ans)

2. How - - - - - - - - - - - - repeated?

4 digit no.s when no. of repitition is allowed

$= 9 \times 9 \times 8 \times 7$

$= 4536$ ways (Ans)

3. How - - - - - - - - - - - repeated

Total digits = 6

3 digit even no. can be made with repitition allowed.

$= 5 \times 4 \times 3$

$= 60$

$$^3P_1 = \frac{3!}{(3-1)!} = \frac{3!}{2} = \frac{3 \times 2!}{2!} = 3$$

$$^5P_3 = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = 60 \text{ (Ans)}$$

4. Find - - - - - - - - - - - - even

$5 \times 4 \times 3 \times 2 = 120$ ways

Possibilities of these will be even $= \frac{2}{5} = 0.4$

$= 120 \times 0.4$

$= 48$ (Ans)

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Q5- From - - - - - - - - position

$$^8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8 \times 7 \times \cancel{6!}}{\cancel{6!}} = 56 \text{ (Ans)}$$

Q6- Find n if $^{n-1}P_3 : {}^nP_4 = 1:9$

$$\frac{(n-1)!}{(n-1-3)!} = \frac{1}{9}$$

$$\frac{n!}{(n-4)!}$$

$$\frac{(n-1)!}{(n-4)!} = \frac{1}{9}$$

$$\frac{n!}{(n-4)!}$$

$$\frac{\cancel{(n-4)!}\,(n-1)!}{\cancel{(n-4)!}\;n!} = \frac{1}{9}$$

$$\frac{1}{n} = \frac{1}{9}$$

$$n = 9 \text{ (Ans)}$$

7. Find r if (i) $5P_r = 2^6P_{r-1}$

$$\frac{5!}{(5-r)!} = \frac{2 \times 6!}{[6-(r-1)]!}$$

$$\frac{5!}{(5-r)!} = \frac{2 \times 6!}{(7-r)!}$$

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$$\frac{5!}{(5-r)!} = \frac{2 \times 6!}{(7-r)(6-r)(5-r)!}$$

$$\frac{5!}{} = \frac{2 \times 6!}{(7-r)(6-r)}$$

$$(7-r)(6-r) = \frac{2 \times 6!}{5!}$$

$$(7-r)(6-r) = \frac{2 \times 6 \times 5!}{5!}$$

$$(7-r)(6-r) = 12$$

$$42 - 13r + r^2 = 12$$

$$r^2 - 13r + 42 - 12 = 0$$

$$r^2 - 13r + 30 = 0$$

$$r^2 - 10r - 3r + 30 = 0$$

$$r(r-10) - 3(r-10) = 0$$

$$(r-10)(r-3) = 0$$

$r = 10$ $\quad$ $\boxed{r = 3}$ Ans

(iii) ${}^5P_r = {}^6P_{r-1}$

$$\frac{5!}{(5-r)!} = \frac{6!}{6-(r-1)!}$$

$$\frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$$

$$\frac{5!}{(5-r)!} = \frac{6!}{(7-r)(6-r)(5-r)!}$$

$$5! = \frac{6!}{(7-r)(6-r)}$$

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$(7-r)(6-r) = \frac{6 \times 5!}{5!}$

$(7-r)(6-r) = 6$

$r^2 - 13r + 42 - 6 = 0$

$r^2 - 9r - 4r + 36 = 0$

$r(r-9) - 4(r-9) = 0$

$(r-9)(r-4) = 0$

$r = 9$, $\boxed{r = 4}$

Q8 - How - - - - - - - - - - - - once

Total digits = 8 [Equation]

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320$ Ans)

Q9 - How - - - - - - - - - - - if

a) 4 letters are used at a time

$^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$ (Ans)

b) all letters are used at a time

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$ (Ans)

c) all letters are used but first letter is a vowel

first letter must be vowel then no. of words =

$^2P_1 \times {^5P_5} = 2 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 240$

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Q10- MISSISSIPPI , I = 4 , S = 4 P = 2

No. of total words = $\frac{11!}{4! \times 4! \times 2!}$

$= \frac{11 \times 10 \times 9 \times \overset{4}{8} \times 7 \times \overset{3}{6} \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1 \times 2 \times 1} = 34650$

When I come together :- (IIII) SSSS MPP

No. of words = $\frac{8!}{4! \times 2!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2 \times 1} = 840$

No. of words when I does not come together :-

$= 34650 - 840$

$= 33810$ (Ans)

Combination :- (i) $^nC_r = \frac{n!}{r!(n-r)!}$

$\therefore {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{{}^nP_r}{r!}$

(ii) $\boxed{r!\ {}^nC_r = {}^nP_r}$

(iii) ${}^nC_x = {}^nC_y$

$\Rightarrow \boxed{x + y = n}$

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(iii) $\boxed{{}^nC_r = {}^nC_{n-r}}$

$${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)![n-(n-r)]!}$$

$$= {}^nC_{n-r}$$

$${}^nC_x = {}^nC_y$$

$${}^nC_{n-x} = {}^nC_y$$

$$\Rightarrow n-x = y$$

$$\Rightarrow n = x+y$$

☆ $\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

L.H.S.

$${}^nC_r + {}^nC_{r-1}$$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

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$$\frac{(n+1)\times n!}{r(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!} = \frac{(n+1)!}{r!(n+1-r)!}$$

$$= {}^{n+1}C_r = R.H.S.$$

$L.H.S. = R.H.S.$ Hence proved

Exercise = 7.4

Q1- If ${}^nC_8 = {}^nC_8$ find ${}^nC_2$

$n-2=8$

$n=10$

$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-8)!}$

${}^{10}C_2 = \frac{10\times 9\times 8!}{2\times 1\times 8!} = 45$ (Ans)

Q2- (i) Determine n if ${}^{2n}C_3 : {}^nC_3 = 12:1$

$\frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$

$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$

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$$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\frac{\frac{2n \times (2n-1) \times (2n-2) \times (2n-3)!}{3!(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!(n-3)!}} = \frac{12}{1}$$

$$\frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)(n-3)}{3!(n-3)!}} = \frac{12}{1}$$

$$\frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\frac{2n \times (2n-1) \times 2(n-1)}{n(n-1)(n-2)} = \frac{12}{1}$$

$$\frac{4 \times n \times (2n-1)}{n \times (n-2)} = \frac{12}{1}$$

$$\frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

Simplifying

$$4 \times (2n-1) = 12 \times (n-2)$$

$$8n - 4 = 12n - 24$$

$$12 - 8n = 24 - 4$$

$$\boxed{n = 5}$$

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(ii) Given $^{2n}C_3 : {}^nC_3 = 11:1$

$$\frac{^{2n}C_3}{^nC_3} = \frac{11}{1}$$

$$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{11}{1}$$

$$\frac{\frac{2n \times (2n-1)(2n-2)(2n-3)!}{3!(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!(2n-3)!}} = \frac{11}{1}$$

$$\frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)}{3!}} = \frac{11}{1}$$

$$\frac{2n \times (2n-1) \times (2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$$

$$\frac{2n \times (2n-1) \times 2(n-1)}{n(n-1)(n-2)} = \frac{11}{1}$$

$$\frac{4 \times n \times (2n-1)}{n \times (n-2)} = 11$$

$$\frac{4 \times 2n-1}{n-2} = \frac{11}{1}$$

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$$4 \times (2n-1) = 11 \times (2n-2)$$
$$8n - 4 = 11n - 22$$
$$11n - 8n = 22 - 4$$
$$\boxed{n = 6}$$

Q3- How - - - - - - - - - - - - - circle

$$^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21 \times 20 \times 19!}{2! \times 19!} = \frac{21 \times 20}{2 \times 1} = \frac{420}{2} = 210 \text{ (Ans)}$$

Q4- In - - - - - - - - - - - girls

$$^5C_3 \times {}^4C_3$$

$$\frac{5!}{3!(5-3)!} \times \frac{4!}{3!(4-3)!} = \frac{5!}{3! \times 2!} \times \frac{4!}{3! \times 1!}$$

$$^5C_3 \times {}^4C_3 = 10 \times 4 = 40 \text{ ways}$$

Q5- Find - - - - - - - - - - - - - - - colour

$$^6C_3 \times {}^5C_3 \times {}^5C_3$$

$$\frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!}$$

$$\frac{6!}{3! \times 3!} \times \frac{5!}{3! \times 2!} \times \frac{5!}{3! \times 2!}$$

$$\frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} \times \frac{5 \times 4 \times 3!}{3! \times 2!} \times \frac{5 \times 4 \times 3!}{3! \times 2!}$$

$$\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1} \times \frac{5 \times 4}{2 \times 1}$$

$$= 2000 \text{ (Ans)}$$

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6. Determine - - - - - - - - - - - - combination

$$^4C_1 \times {}^{48}C_4 = \frac{4!}{1!(4-1)!} \times \frac{48!}{4!(48-4)!}$$

$$= \frac{4!}{1! \times 3!} \times \frac{48!}{4! \times 44!}$$

$$\frac{4 \times 3 \times 2 \times 1}{1 \times 3 \times 2 \times 1} \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{4! \times 44!} = 4 \times 194580$$

$$= 778320 \text{ (Ans)}$$

7. In - - - - - - - - - - - - - bowlers

$$^5C_4 \times {}^{12}C_7 = \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!}$$

$$= \frac{5!}{4! \times 1!} \times \frac{12!}{7! \times 5!}$$

$$= 3960 \text{ (Ans)}$$

8. A bag - - - - - - - - - - - - - - - selected

$$^5C_2 \times {}^6C_3 = \frac{5!}{2!(5-2)!} \times \frac{6!}{3!(6-3)!} = \frac{5!}{2! \times 3!} \times \frac{6!}{3! \times 3!}$$

$$\frac{5 \times 4 \times 3!}{2 \times 3!} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1} = 200 \text{ (Ans)}$$

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9. In - - - - - - - - - - - - - - - - Student?

$$^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \times 4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!}$$

$$= \frac{210}{6} = 35 \text{ (Ans)}$$

Miscellaneous Exercise

1. How - - - - - - - - - - - - - - - - DAUGHTER?

$^3C_2 \times {^5C_3}$

$$\frac{3!}{2!(3-2)!} \times \frac{5!}{3!(5-3)!} = \frac{3!}{2!1!} \times \frac{5!}{3!2!}$$

$$= 30$$

$^5P_5$

$$\frac{5!}{(5-5)!} = \frac{5!}{0} = 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120$$

$$= 120 \times 30 = 3600 \text{ (Ans)}$$

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Q: How - together.

EQUATION = 8 letters

EQUIO = Vowels QTN = letters

Since the vowels and consonants occur together, we assume the 5 vowels as one object and the 3 consonants as another object.

These two objects can be arranged among themselves in $2! = 2$ ways

5 vowels can Interchange places in $5!$ ways

3 Consonants can Interchange places in $3!$ ways

Total no. of words

$= 2! \times 5! \times 3!$

$= 1440$ words (Ans)

3. A committee - of

(i) exactly 3 girls?

No. of committee with exactly 3 girls $= {}^9C_4 \times {}^4C_3$

$\Rightarrow \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 4$

$= 12 \times 7 \times 6$

$= 504$ ways (Ans)

(ii) atleast 3 girls?

No. of committe atleast 3 girls $= {}^9C_4 \times {}^4C_3 + {}^4C_3 + {}^4C_4$

$\Rightarrow \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 4 + \frac{9 \times 8 \times 7}{3 \times 2} \times 1$

$= 504 + 12 \times 7$

$= 504 + 84 \Rightarrow 588$ (Ans)

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(iii) almost 3 girls?

No. of commitee with atmost 3 girls =

$$({}^{9}C_4 \times {}^{4}C_3) + ({}^{9}C_5 \times {}^{4}C_2) + ({}^{9}C_6 \times {}^{4}C_1) + ({}^{9}C_7 \times {}^{4}C_0)$$

$$504 + \left[\frac{\overset{3}{\cancel{9}} \times 8 \times 7 \times \overset{4}{\cancel{6}}}{\cancel{4} \times 3 \times \cancel{2}} \times \frac{\overset{2}{\cancel{4}} \times 3}{\cancel{2}}\right] + \left[\frac{9 \times 8 \times 7}{3 \times 2} \times 4\right] + \left[\frac{9 \times 8}{2}\right]$$

$$504 + (9 \times 4 \times 7 \times 3) + (12 \times 7 \times 4) + 36$$

$$504 + 756 + 336 + 36$$

$$1260 + 372$$

$$\boxed{1632}$$ Ans

4. If - E?

Total letters = 11

The required no. of words before the first word starting with E = no. of words which begin with A because only A appears before E in the above alphabetical order. When A is fixed in the first place, we have to arrange the remaining 10 letters in which there are two I's and two N's.

The remaining no. of words = $\frac{10!}{2! \times 2!}$

$$\Rightarrow \frac{\overset{5}{\cancel{10}} \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2} \times 1}$$

$$\Rightarrow 5040 \times 30 \times 6$$

$$\Rightarrow 5040 \times 180$$

$$\Rightarrow 907200$$ (Ans)

5. How - repeated?

No's divisible by 10 must have '0' in its unit's place.

The remaining 5 digits can be arranged in the remaining 5 vacant places in ${}^{5}P_5 = 5!$ ways

$= 5 \times 4 \times 3 \times 2 \times 1 \Rightarrow 20 \times 6 \Rightarrow 120$ ways (Ans)

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Ques 6- The English - - - - - - - - - - - - - - - alphabet?

2 vowels out of 5 can be selected in $5C_2$ ways

$$\frac{5\times4}{2\times1} = 10 \text{ ways}$$

2 Consonants out of 21 can be selected in $21C_2$

$$\frac{21\times20}{2\times1} = 210 \text{ ways}$$

The no. of selections of 2 vowels and consonants in

$10\times210 = 2100$

2100 selections has 4 letters which can be arranged among themselves in $4P_4 = 4! = 24$ ways

Required no. of different words $= 2100\times24$

$= \boxed{50400 \text{ words}}$ Ans

7. In - - - - - - - - - - - - - - - questions

The possibilities of selecting 8 questions are

Part I (5)	Part II (7)
(i) 3	5
(ii) 4	4
(iii) 5	3

The required no. of ways

$(5C_2\times7C_5)+(5C_4\times7C_4)+(5C_5\times7C_3)$

$\left[\frac{5\times4}{2}\times\frac{7\times6}{2}\right]+\left[\frac{5\times7\times6\times5}{3\times2\times1}\right]+\left[\frac{1\times7\times6\times5}{3\times2\times1}\right]$

$(10\times21)+(5\times35)+(1\times35)$

$= 420$ ways (Ans)

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Q8- Determine - - - - - - - - - - - - - - - King

4 Kings and 48 other cards

One King can be selected out of 4 in $^4C_1$ ways and 4 other cards out of 48 in $^{48}C_4$

Total combinations $= {}^4C_1 \times {}^{48}C_4$

$$= \frac{\cancel{4}^{\cancel{2}}}{\cancel{4}} \times \frac{\cancel{48} \times 47 \times \cancel{46}^{23} \times \cancel{45}^{15}}{\cancel{4} \times \cancel{3} \times \cancel{2}}$$

$\Rightarrow 778320$ ways

Ques- It is - possible?

Second, fourth, sixth and the eighth places are even, women can be arranged in 4 even places in $^4P_4 = 4!$ ways

5 men can be arranged in 5 odd places in $^5P_5 = 5!$ ways

Total arrangements $= 4! \times 5!$

$= 24 \times 210$

$= 2880$ ways

Ques- From - Chosen?

There are two possibilities -

(i) Particular 3 Students join

(ii) Particular 3 students do not join

In first case of inclusion, we have to choose 7 more students out of remaining 22 student in $^{22}C_7$ ways

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In second case of exclusion, we have to choose all 10 students out of remaining 22 students in $^{22}C_{10}$ ways.

Required no. of ways $= {}^{22}C_7 + {}^{22}C_{10}$

$= 170544 + 646646$

$= 817190$ ways

Ques 11 In how - - - - - - - - - - - - - - together

ASSASSINATION = 13 letters :

A = 3 times S = 4 times

I = 2 times N = 2 times

Since all the S's are to occur together, we take as a single object (SSSS). This single object together with 9 remaining letters become 10 objects (SSSS) AAAIINNTO which can be arranged in $\frac{10!}{3!2!2!}$ ways

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 1 \times 2 \times 1}$

$= 5040 \times 30$

$= 151200$ (Ans)